AN IMPLEMENTATION OF RESIDUAL CAPACITY INEQUALITIES

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1. Introduction

Residual Capacity Inequalities have been introduced by Magnanti, Mirchandani and Vachani [3] in the context of Network Design. Later Atamturk and Rajan [1] gave an exact separation algorithm for them. These inqualities were particularly useful in a project on capacity planning, see [2]. For that reason we decided to add them to the CGL library of COIN-OR.

2. Residual Capacity Inequalities

Consider the set

$$Q = \{x \in \mathbb{R}^N,\ y \in \mathbb{Z}^+ \ :\ \sum_{i \in N} a_i x_i \le a_0 + y,\ 0 \le x \le 1\}.$$

Magnanti et al. [3] showed that the convex hull of Q is obtained by adding the inequalities below to the linear relaxation of Q.

$$\sum_{i \in S} a_i(1 - x_i) \ge \rho(\eta - y),\ S \subseteq N. \tag{1}$$

Here $\eta = \lceil \sum_{i \in S} a_i - a_0 \rceil$ and $\rho = \sum_{i \in S} a_i - a_0 - \lfloor \sum_{i \in S} a_i - a_0 \rfloor$. These are called *residual capacity inequalities* (RCI).

Atamturk et al [1] showed that given a point (x, y) in the linear relaxation of Q one can find a violated RCI inequality, if there is any, by letting $S = \{i \in N \ :\ x_i > y - \lfloor y \rfloor\}$.

We have implemented a procedure that given a mixed integer linear program, it scans the matrix looking for inequalities of the type

$$\sum_{i \in N} a_i x_i \le b + d \sum_{j \in M} y_j$$

with

$$0 \le x_i \le u_i$$
$$y_j \in \mathbb{Z}^+.$$

By treating $\sum_{j \in M} y_j$ as one integer variable, and doing a linear transformation of the variables x, we obtain an inequality as in the definition of the set Q. Then the separation algorithm can be applied. The upper bounds u_i should be as tight as possible, this influences the quality of the cutting planes found.

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3. Experiments

Below we present some experiments with some instances from [2]. We give the name of the instance, then the value of the linear program, then the lower bound obtained by adding only RCI inequalities, and finally the lower bound given by adding only mixed integer rounding inequalities (MIR). The separation code for RCI is in `CglResidualCapacity.cpp`. The code that produces MIR inequalities is in `CglMixedIntegerRounding.cpp`.

name	LP value	RCI	MIR
`capPlan1`	677.43	773.87	765.62
`capPlan2`	258.90	320.85	302.19
`capPlan3`	119.46	182.86	177.17
`capPlan4`	296.57	457.40	449.87
`capPlan5`	317.94	357.58	349.55
`capPlan6`	234.33	299.97	283.92
`capPlan7`	196.038	245.245	244.67

Table 1. Experiments with capacity planning instances

References

[1] A. Atamtürk and D. Rajan, *On splittable and unsplittable flow capacitated network design arc-set polyhedra*, Math. Program., 92 (2002), pp. 315–333.

[2] F. Barahona, S. Bermon, O. Günlük, and S. Hood, *Robust capacity planning in semiconductor manufacturing*, Naval Res. Logist., 52 (2005), pp. 459–468.

[3] T. L. Magnanti, P. Mirchandani, and R. Vachani, *The convex hull of two core capacitated network design problems*, Math. Programming, 60 (1993), pp. 233–250.